

# Sensitivity Analysis for Acid Leaching of Copper Oxides

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## Abstract

In this study, a sensitivity analysis to variations in chemical parameters present in a copper extraction transport model, from a copper oxide ore bed, was performed. Specifically, a two-dimensional solute transport model was utilized, which was previously validated by authors with experimental data. The mathematical model consists of a transport equation for the concentration of sulfuric acid in the liquid phase, a transport equation for the concentration of copper in the liquid phase, and a transport equation for copper in the solid phase. Chemical analysis of the results from computational experiments provides relevant information regarding the leaching process.

*Keywords:* copper, leaching, sensitivity, modeling.

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## 1. Introduction.

Leaching is a mass transfer process between the leaching solution (fluid phase) and the ore bed (solid phase) [12, 16]. The heap leaching process used for copper production can be considered as a *flow of two fluid phases* (liquid and gaseous) and a *component transport* (sulfuric acid, copper ions, water, air, or oxygen, for example) in a porous medium (the heap) [8, 15, 17, 20]. Therefore, the two phenomena of interest are the fluid flow and the physical-chemical reactions. This paper follow the *decoupled approach* where both of these phenomena were studied separately if the extent of leaching did not influence the flow pattern. In other words, the flow pattern in a heap only depends on the initial conditions of the heap. The fluid flow in heap leaching, the first problem, has been studied in [4, 5, 15, 19]. Specifically, Cariaga et.al. [4], studied the application of the classical two phase flow equations (see [9, 13], for a general study of these equations) in heap leaching. On the other hand, the second problem, the component transport, has been studied in [3, 15, 16, 19]. Specifically, Cariaga et.al. in [3] and Li in [15] studied this problem in the two phase context, but without theoretical convergence analysis. Only the component transport is considered in this paper.

This work is a continuation of [3, 7, 16, 19]. It is also a continuation of [16] where the porous medium is saturated with the leach solution. In [19] a mathematical model similar to that used in this article is presented, but the differential equations are solved in one dimension with a different numerical method. A sensitivity analysis is also performed in [19] but under more restrictive conditions. In [3] a generalized version to the model in [16, 19] was presented in the context of multi-component transport and two-phase

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9 flow in porous media. Moreover in [3] the model is two-dimensional and  
10 applicable under two-phase flow conditions. Finally in [7] a full convergence  
11 and theoretical analysis of the mathematical model was performed.  
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15 The mathematical model utilized in this work was validated with experi-  
16 mental data from [16, 19]. Also, the methodology used in this work is based  
17 on the realization of computational experiments, where the numerical val-  
18 ues of the parameters used in the simulations in the Chilean copper mining  
19 simulations and in scientific articles were already published. Specifically, the  
20 methodology consists of varying one parameter at a time, as has been done,  
21 in similar studies such as [11] and [18].  
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25 The efficiency of the leaching process can be measured by the ratio of the  
26 mass of extracted copper and the mass of the irrigated acid. Such efficiency  
27 depends on several physicochemical parameters, such as: mineral chemistry,  
28 the availability of copper, and the porosity of the leach pile. It is on the  
29 basis of these characteristics that the optimal volume and the concentration  
30 of the sulfuric acid must be determined. In this context, a sensitivity analysis  
31 evaluates the influence of the relevant physicochemical parameters on the  
32 efficiency of the leaching process and thus achieving the optimum. For this  
33 reason, transport models can be an excellent design and optimization tool.  
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37 In the present work, a sensitivity analysis is performed using the transport  
38 model proposed in [7]. The main goal is to evaluate the influence of relevant  
39 chemical parameters on the efficiency of the cupric oxide leaching process,  
40 using a mathematical model as an optimization and design tool.  
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44 Additionally, a miscalculation at the design stage can lead, for example,  
45 to using too much acid or not achieving an adequate recovery of copper. The  
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nature of the stack material component greatly influences both the leaching kinetics and thus affecting the time required to complete the process and the quantity of the required leaching agent (acid). These variables highly impact the economic and technical evaluation of the process. Accordingly, the design stage requires a tool that can quantify the influence of various physicochemical parameters on the performance process.

The paper is organized in five sections: in section 2, the transport model is presented. In section 3, the materials and methods are presented. In section 4 the results and discussion are presented. Finally, in section 5 the main conclusions are developed and presented.

## 2. Transport Model

The heap is assumed as isothermal and as a non-deformable porous media, formed by two phases -liquid and gaseous-, and two components: sulfuric acid in the liquid phase and copper in both the liquid and solid phase. Figure 1 illustrates the main phase transitions in the heap from a conceptual point of view.

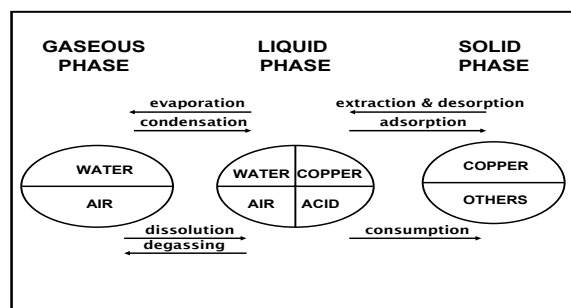
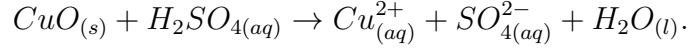


Figure 1. Transition Phase.

In this paper, only oxide minerals are considered which in general terms have the following dissolution chemical reaction [12, 16, 19],



According to the stoichiometry of the reaction, 0.65[kg] of copper would be extracted for each 1[kg] of sulfuric acid. However, the actual yield is much lower since there are a variety of other reactions that occur between the leaching agent and the other species in the ore grains. The kinetics of the reaction depends on the grain size and the chemical nature of both the copper ore and the other ore species that make up the rock.

Following the macroscopic porous media theory, there is a REV (*representative elementary volume*) [1, 13]. The porosity  $\phi$  of the porous medium is defined as the ratio between the *volume of the pore space within the REV* and *the volume of REV* and the saturation of phase  $\alpha = w, n$  where  $\alpha = w$  is the liquid phase,  $\alpha = n$  is the gaseous phase, and  $s_\alpha$  is defined as the ratio between the *volume of fluid  $\alpha$  within the REV* and the *volume of the pore space within the REV*. The customary property that the fluids fill the volume:  $s_w + s_n = 1$  is used. Also, the mass balance must be specified for each component. The mass balance equations for the transport of component  $\kappa$  ( $\kappa = a$  for the sulfuric acid, and  $\kappa = c$  for the copper) in  $\alpha$ -phase can be written as [13]:

$$\frac{\partial (\theta_\alpha c_\kappa^\alpha)}{\partial t} + \nabla \cdot (c_\kappa^\alpha \mathbf{v}_\alpha - \mathbf{D}(\mathbf{v}_\alpha) \nabla c_\kappa^\alpha) + \Phi_\alpha = 0, \quad (1)$$

where  $c_\kappa^\alpha$  is the volumetric concentration defined as the ratio between the *mass of component  $\kappa$  in phase  $\alpha$*  and the *volume of phase  $\alpha$* ,  $\theta_\alpha := \phi s_\alpha$  is

the liquid content,  $\mathbf{v}_\alpha$  is the *Darcy's flow of phase  $\alpha$* ,  $\Phi_\alpha$  [ $kg/m^3 \cdot s$ ] is the irreversible rate of solute removed added to or from the liquid solution,  $\mathbf{D}$  is the dispersion-diffusion tensor given by  $D_{ij} := \alpha_L |\mathbf{v}_\alpha| \delta_{ij} + (\alpha_L - \alpha_T) \frac{u_i u_j}{|\mathbf{v}_\alpha|} + D_m \delta_{ij}$ , where  $\alpha_L$  and  $\alpha_T$  are the longitudinal and transverse dispersities, respectively,  $\delta_{ij} = 1$  if  $i = j$  and  $\delta_{ij} = 0$  if  $i \neq j$ , and  $D_m$  is the molecular diffusion coefficient, which is defined as  $D_m = 0$  in this paper.

On the other hand, if it is assumed that there is an isotherm between the liquid phase and the solid phase  $\varphi_\kappa^\alpha = \varphi_\kappa^\alpha(c_\kappa^\alpha)$ , defined as the ratio between the *mass of component in the solid phase* and the *mass of the solid phase*, then using the assumption that sorption only occurs from the liquid to the solid phase, the equation for the liquid phase can be modified to include adsorption:

$$\frac{\partial (\phi_s \rho_s \varphi_\kappa^\alpha)}{\partial t} + \frac{\partial (\theta_\alpha c_\kappa^\alpha)}{\partial t} + \nabla \cdot (c_\kappa^\alpha \mathbf{v}_\alpha - \mathbf{D}(\mathbf{v}_\alpha) \nabla c_\kappa^\alpha) + \Phi_\alpha = 0, \quad (2)$$

where  $\phi_s := 1 - \phi$ ,  $\rho_s$  is the bulk ore density. Note that the most common sorption isotherms [14] are: *Langmuir* type  $\varphi_\kappa^\alpha(w) := \frac{k_1 w}{1 + k_2 w}$ , *Freundlich* type  $\varphi_\kappa^\alpha(w) := k_3 w^p$ ,  $0 < p$ , and *mixed* type  $\varphi_\kappa^\alpha := \frac{k_4 w^p}{1 + k_5 w^p}$ .

**Remark 1.** Assuming that the chemical processes do not affect the flow, the mass balance equations for the two phases are completely decoupled and can be solved first. In the general transport equation, (1) it is assumed that for  $\alpha = w, n$  the functions  $\theta_\alpha = \phi s_\alpha$  and  $\mathbf{v}_\alpha$  are known. For a detailed analysis of the classical two phase flow equations, in the context of heap leaching operations, refers to Cariaga [2].

**Remark 2.** For the basic chemical reaction defined in this article, it is sufficient to consider just the unsaturated flow. That is, the fluid flow problem is simplified enough to solve the Richard's equation (make  $p_n$  equal to a constant on  $\Omega \times (0, T)$  in the two phase flow system [6, 13].

### 2.1. Sulfuric acid transport equation

Following the description of Muñoz et al. [16], during the migratory process through the porous medium, the sulfuric acid ( $H_2SO_4$ ) reacts with the various ore minerals, including copper. This reaction is an irreversible process that generates a consumption (neutralization) of the acid present in the liquid solution. The acid consumption depends on the composition of the host ore and overlying mineralized grains. Copper ores located in basalt and diabase rocks have a high consumption of acid in monzonite, andesite, and granite, a moderate consumption amount in quartz-sericite, and a small consumption, close to the stoichiometric consumption of acid, in quartz and sandstone.

From equation(1), with  $\kappa = a$  for the sulfuric acid and  $\alpha = w$  for the liquid phase, it is assumed that the acid consumption velocity is proportional to the sulfuric acid concentration, that is,  $\Phi_\alpha := \theta_w \mu c_a^w$ . Therefore, the transport equation of sulfuric acid in the leach solution is given by

$$\frac{\partial(\theta_w c_a^w)}{\partial t} + \nabla \cdot (c_a^w \mathbf{v}_w - \mathbf{D}(\mathbf{v}_w) \nabla c_a^w) + \theta_w \mu c_a^w = 0, \quad (3)$$

where  $\mu$  is a first-order reaction constant (consumption factor) and  $c_a^w$  is the volumetric concentration of sulfuric acid in the leach solution.

## 2.2. Copper transport equation in liquid phase

Following the description used by Muñoz et al. [16], the movement of copper through the porous media is governed by two processes. The first process consists of a chemical reaction between the leaching agent and the mineral particles, where copper, present in the grains of soil, is converted into soluble particle, such as copper sulfate, and thus passing from the solid to the liquid state (extraction). The second process involves a copper adsorption/desorption phenomenon, which takes place as the copper-bearing solution travels through the heap. First order kinetics are involved and describe the interaction that occurs between the leaching agent ( $H_2SO_4$ ) and the mineral particle ( $Cu$ ). Therefore, from (2) with  $\Phi_\alpha := \phi_s \rho_s k_e c_a^w c_c^s$ , the transport equation of copper in the leach solution is given by

$$\frac{\partial(\phi_s \rho_s \varphi_c^w(c_c^w))}{\partial t} + \frac{\partial(\theta_w c_c^w)}{\partial t} + \nabla \cdot (c_c^w \mathbf{v}_w - \mathbf{D}(\mathbf{v}_w) \nabla c_c^w) - \theta_w \rho_s k_e c_a^w c_c^s = 0, \quad (4)$$

where  $k_e$  is a first-order kinetic constant including information regarding the particle and its reaction,  $c_c^s$  is the concentration of copper associated with the solid phase (cf.(5)), and  $c_c^w$  is the volumetric concentration of copper in leach solution.

## 2.3. Copper transport equation in solid phase

Following the description of [16], the change in the copper concentration in the solid phase is given by

$$\frac{\partial c_c^s}{\partial t} + \theta_w k_e c_a^w c_c^s = \frac{\partial(\phi_s \varphi_c^w(c_c^w))}{\partial t}, \quad (5)$$

where  $c_c^s$  is the concentration of copper in the solid phase, *i.e.*,

$$c_c^s := \frac{\text{mass of copper in solid phase}}{\text{mass of solid phase}}.$$



#### 2.4. Initial and boundary conditions

It is considered a 2D geometry, i.e., a transversal cut of the heap (cf. Figure 2). The boundary of  $\Omega \subset \mathbb{R}^2$ , i.e.,  $\partial\Omega$  is expressed as  $\partial\Omega = \Gamma^i \cup \Gamma^o \cup \Gamma^t$ , where  $\Gamma^i$  is the input boundary (zone of irrigation),  $\Gamma^o$  is the output boundary (zone of drainage), and  $\Gamma^t$  is the atmospheric boundary:

$$\begin{aligned}
 (c_a^w \mathbf{v}_w - \mathbf{D} \nabla c_a^w) \cdot \mathbf{n} &= c_a^i \mathbf{v}_w \cdot \mathbf{n} & \mathbf{x} \in \Gamma^i & \quad t \geq 0 \\
 (c_{a,c}^w \mathbf{v}_w - \mathbf{D} \nabla c_{a,c}^w) \cdot \mathbf{n} &= 0 & \mathbf{x} \in \Gamma^t & \quad t \geq 0 \\
 (c_{a,c}^w \mathbf{v}_w - \mathbf{D} \nabla c_{a,c}^w) \cdot \mathbf{n} &= (c_{a,c}^w - c_{a,c}^o) \mathbf{v}_w \cdot \mathbf{n} & \mathbf{x} \in \Gamma^o & \quad t \geq 0 \\
 (c_c^w \mathbf{v}_w - \mathbf{D} \nabla c_c^w) \cdot \mathbf{n} &= 0 & \mathbf{x} \in \Gamma^i & \quad t \geq 0 \\
 c_{a,c}^w(\mathbf{x}, t) &= c_{a,c}^o & \mathbf{x} \in \Omega & \quad t = 0 \\
 c_c^s(\mathbf{x}, t) &= c_c^{so} & \mathbf{x} \in \Omega & \quad t = 0,
 \end{aligned} \tag{6}$$

where, as usual,  $\mathbf{n}$  is the normal unit vector to  $\partial\Omega$ , outward to  $\Omega$ ,  $c_{a,c}^i$  the concentration of sulfuric acid and copper in the irrigation solution,  $c_c^{so} := \lambda_c^s G_c^s$ , where  $\lambda_c^s$  is the leachable fraction of the total copper contained in the heap and  $G_c^s$  is the grade of the ore.

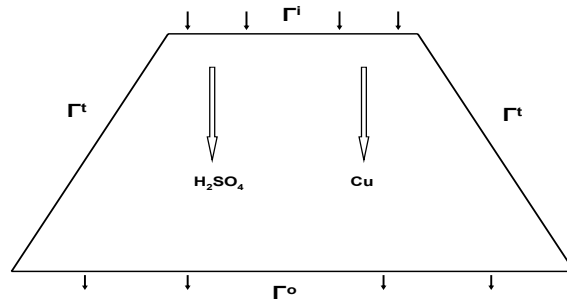


Figure 2. Mathematical domain:  $\Omega$ .

### 3. Material and Methods

The system (3)-(4)-(5), under initial and boundary conditions given by (6), was solved in a two-dimensional space domain, with a finite volume numerical scheme. For a mathematical full description and the respective theoretical convergence analysis see [7].

In these simulations only one isotherm is considered for the copper in the liquid phase and is given by  $\varphi_2 = k_d c_c^w$ . Table 1 contains a full description of the utilized parameters (see [7]).

Name	Symbol	Value	Unit
irrigation rate	$R$	18	$[l/hr/m^2]$
velocity	$\mathbf{u}_w$	$(18/36) \cdot 10^{-5}$	$[m/s]$
water saturation	$s_w$	0.7	$[-]$
porosity	$\phi$	0.33	$[-]$
consumption factor	$\mu$	$10^{-5}$	$[1/s]$
kinetic constant	$k_e$	$8.3 \cdot 10^{-7}$	$[m^3/kg \cdot s]$
equilibrium distribution constant	$k_d$	$8.67 \cdot 10^{-5}$	$[m^3/kg]$
bulk density	$\rho_s$	1800	$[kg/m^3]$
leachable fraction	$\lambda_c^s$	0.62	$[-]$
grade of the ore	$G_c^s$	$2.9 \cdot 10^{-3}$	$[kg/kg]$
initial acid concentration	$c_a^o$	10	$[kg/m^3]$
initial copper concentration	$c_c^o$	0	$[kg/m^3]$
top acid concentration	$c_c^o$	40	$[kg/m^3]$
heap width	$W$	25	$[m]$
heap height	$H$	5	$[m]$

Table 1. Numerical values for physical parameters.

The methodology is based on the realization of computational experiments in which the numerical values of the parameters used in the simulations from Chilean copper mining and scientific articles have already been published. Specifically, the methodology is to vary one parameter at a time, as has been done in similar studies such as [11] and [18].

## 4. Results and Discussion

### 4.1. Reference Solutions

In figures 3, 4 and 5 (which are reproduced from [6]), the evolution of the concentration is shown for 19 days, where P1 is a generic point on  $\Gamma^i$ , P2 is in the middle of  $\Omega$ , and P3 is on  $\Gamma^o$ , (see figure 2).

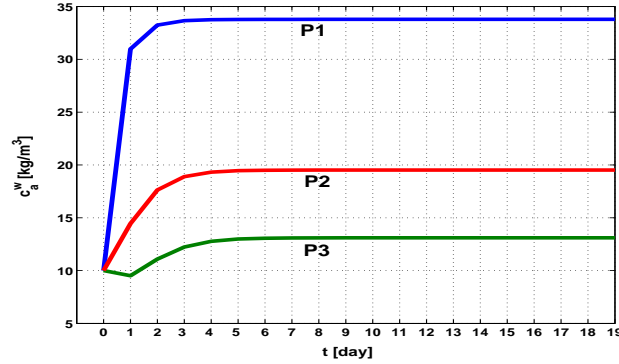


Figure 3. Sulfuric Acid in Liquid Phase:  $c_a^w$ .

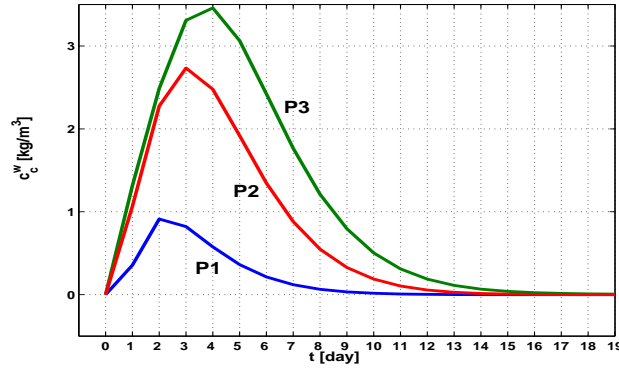


Figure 4. Copper in Liquid Phase:  $c_c^w$ .

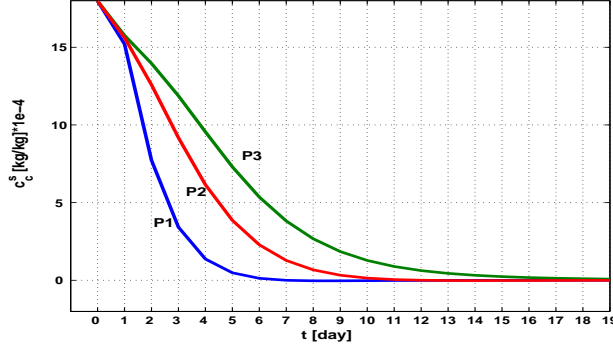


Figure 5. Copper in the Solid Phase:  $c_c^s$ .

Figure 3 shows that the concentration of sulfuric acid varied significantly with the sampling height. This result was expected because as the solution drains, the acid reacts not only with the copper ore (which is at a low level) but also with other species in the ore (carbonates, silicates and oxides and various other metal hydroxides such as: sodium, potassium, calcium and iron).

Once the concentration has reached a steady state, about  $21[kg]$  of acid per  $1[m^3]$  of irrigating solution are consumed from the top to the bottom of the stack.

Also important to note is the acid solution exits out of the stack at concentration ( $13[kg/m^3]$ ) so that the pH equals 0.6. This condition ensures the chemical stability of the cupric ion, the stable ions in solution during the drainage will not return to the solid phase in the form of basic salts.

Figure 4 shows that it would take 14 days to extract the entire amount of available copper in the stack at an irrigation rate of  $18[L/h/m^2]$ . This implies a consumption of  $6.05[m^3]$  of acid solution for every  $[m^2]$  of irrigated

area. If the concentration of the acid solution is  $40[kg]$  of acid for  $1[m^3]$  of solution, then it would take  $242[kg]$  of acid for  $1[m^2]$  of irrigated area.

Moreover, given: the density of the solid phase ( $1800[kg \cdot s/m^3]$ ), the concentration of copper in the solid phase ( $2.9 \cdot 10^{-3}[kg \cdot Cu/kg \cdot s]$ ), and the fraction of available copper (0.62) we can determine that the amount of extractable copper in a column of height  $3[m]$  and a cross section  $1[m^2]$  is equal to  $9.70[kg]$ .

Therefore,  $0.040[kg]$  of copper is recovered by  $1[kg]$  of irrigated acid.

Figure 5 shows that at the bottom of the stack the extraction rate of available copper is less than at the top of the stack. This is explained by the decline experienced in the concentration of the drainage leaching agent.

#### 4.2. Sensitivity Study

In order to test the ability and performance of the model used to simulate the leaching process under different conditions and to provide a basis for the optimization of operation conditions, some sensitivity tests are carried out. In these sensitivity test the parameters of importance are changed one at a time.

In figures 6, 7, 8, 9 and 10, some sensitivity tests are presented. The reference parameters are the initial set defined in the previous section. The considered parameters are: (i) the volumetric concentration of sulfuric acid in the irrigation solution, that is,  $c_a^i$  (see Figures 6, 7 and 8), (ii) the consumption factor, a first-order reaction constant, that is,  $\mu$  (see Figure 9), and (iii) first-order kinetic constant, that is,  $k_e$  (see Figure 10). All these parameters have a major impact on the results of the technical-economic assessment of a leaching process. Indeed, the main input into the process is the amount

of acid being incorporated into the irrigation solution. It is also essential to determine an optimal acid concentration value, i.e. a value that ensures complete extraction of copper available in the stack in an acceptable period. Parameters (ii) and (iii) above, are not subject to adjustment or control during the operation. They depend on the nature and chemical composition of the ore, i.e. substances that make up the gangue and the way the copper mineralization is distributed within the ores. It is vital to know the parameter values at the design stage since they are essential to establish the amount of acid required in the process and the time when the extraction is completed.

In Figures 6-10, the evolution on point P2 is shown, which is a generic point in the middle of heap  $\Omega$ , (see Figure 2), specifically,  $P2 = (6.9[m], 1.6[m])$ .

#### 4.2.1. Influence of irrigation sulfuric acid concentration: $c_a^i$ .

The reference value is  $c_a^i = 40[kg/m^3]$ . In figures 6, 7, and 8, the evolution of sulfuric acid in liquid solution  $c_a^w$ , the copper in liquid solution  $c_c^w$ , and copper in solid phase,  $c_c^s$ , are shown respectively, when the reference value is amplified to  $c_a^i = 60[kg/m^3]$  and reduced to  $c_a^i = 20[kg/m^3]$ .

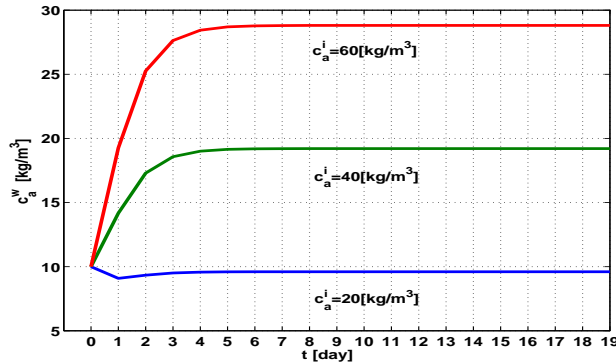


Figure 6. Influence of  $c_a^i$  on  $c_a^w$ .

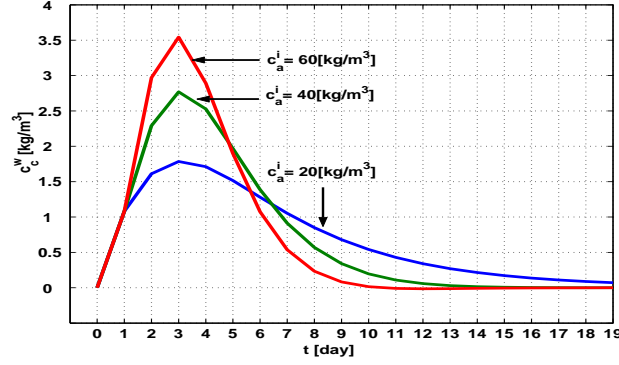


Figure 7. Influence of  $c_a^i$  on  $c_c^w$ .

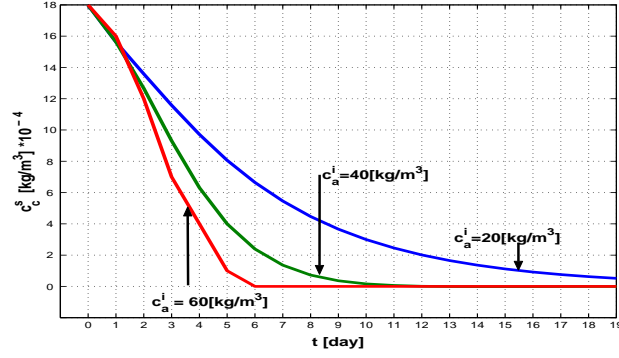


Figure 8. Influence of  $c_a^i$  on  $c_c^s$ .

Figure 6 shows that if the concentration  $c_a^i$  in the irrigation solution increases then the concentration  $c_a^w$  of sulfuric acid in the liquid phase also increases along the heap. Therefore, control over the  $c_a^i$  would allow a control of  $c_a^w$  and, consequently, control over the potential extraction levels of copper.

Figure 7 shows that if the concentration  $c_a^i$  increases then the maximum possible concentration of copper in the liquid phase also increases. Also, it shows that increasing the initial acid concentration from 40 to 60[kg/m³] reduces the time needed for the complete removal of copper from 14 to 10

days. A concentration of  $60[kg/m^3]$  and an irrigation rate of  $18[L/h/m^2]$  would extract  $0.037[kg]$  of copper for every  $1[kg]$  of irrigated acid. Thus, it is necessary to assess if the increase in acid expense is offset by a decrease in processing time.

Figure 8 shows that increasing the concentration  $c_a^i$  implies a greater extraction of copper from the solid phase, i.e., a lower concentration of copper in the solid phase.

Figures 7 and 8 show that if the initial acid concentration decreases from 40 to  $20[kg/m^3]$  that the complete removal of copper in the middle of the pile is not achieved. Based on the results presented in Figure 5, this behavior was expected. Also, a solution of  $20[kg/m^3]$  (which contains less acid than is required by the stack) is already partially neutralized at P2.

Therefore, figures 6, 7, and 8, show that the acid concentration in the irrigation solution, could be a useful control factor with respect to the maximum concentration of copper obtained to be in the liquid phase.

#### 4.2.2. Influence of first order reaction constant: $\mu$ .

The reference value is  $\mu = 10^{-5}[1/s]$ . In figure 9, the evolution of *sulfuric acid in liquid solution*  $c_a^w$  is shown when the reference value is amplified to  $\mu = 10^{-4}[1/s]$  and reduced to  $\mu = 10^{-6}[1/s]$ .



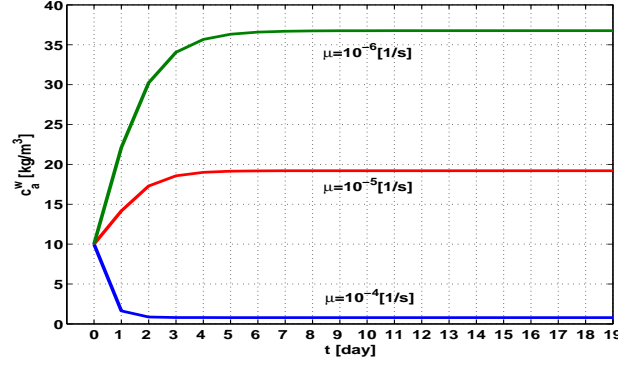


Figure 9. Influence of  $\mu$  on  $c_a^w$ .

Figure 9 shows that if the value of  $\mu$  increases that the level of sulfuric acid in the liquid phase  $c_a^w$  diminishes. Remember that the  $\mu$  parameter should be estimated through experimental observations of the outflow at the base of the heap, and thus. It is not a control parameter. However, once estimated, this parameter could provide information about appropriate values for control parameters.

A low value of  $\mu$  ( $10^{-6}[1/s]$ ) implies that the ore is not very reactive. This feature is desirable because, it means that it increases the fraction of available copper and, also the ratio of copper recovered by  $[kg]$  by the irrigated acid.

In contrast, a high value of  $\mu$  ( $10^{-4}[1/s]$ ) implies that the stone is very reactive (mainly composed of carbonated mineral species). This explains what was observed in Figure 9, the acid concentration in the liquid phase falls to near zero under this condition. It is clear that if  $\mu = 10^{-4}[1/s]$ , then  $c_a^i \geq 40[kg/m^3]$ . Otherwise, the existing copper in the bottom of the stack will not be extracted and the copper recovered at the top of the stack could precipitate in the form of basic salts.

#### 4.2.3. Influence of first order kinetic constant: $k_e$ .

The reference value is  $k_e = 8.3 \cdot 10^{-7} [\text{m}^3/\text{kg} \cdot \text{s}]$ . In figure 10 the evolution of copper in liquid solution  $c_c^w$  is shown when the reference value is reduced to  $k_e = 8.3 \cdot 10^{-8} [\text{m}^3/\text{kg} \cdot \text{s}]$ .

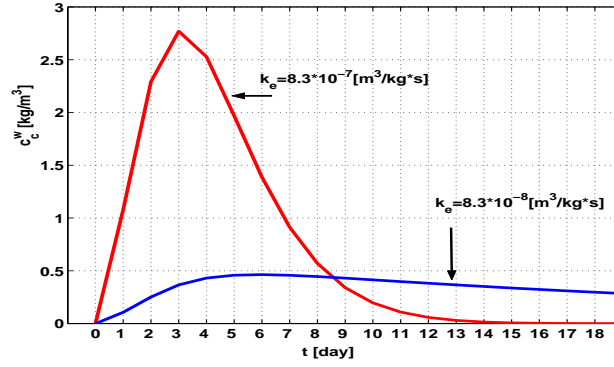


Figure 10. Influence of  $k_e$  on  $c_c^w$ .

The data in Figure 10 show that if the value of  $k_e$  is lower that the concentration of copper in the liquid phase  $c_c^w$  is lower too. The  $k_e$  parameter should be estimated through experimental observations of the outflow at the base of the heap; therefore, it is not a control parameter. However, once estimated it provides information about the appropriate values for the control parameters.

The first order rate constant for the extraction reaction accounts for the time dependency when the available copper in the stack passes to the drainage solution. The  $k_e$  value depends on the chemical nature of the copper ore and how accessible it is to the leaching agent. Indeed, carbonates, sulfates, and copper oxides react rapidly with the sulfuric acid; the reduced oxides (which contain copper and iron in the cuprous ( $\text{Cu}^+$ ) and ferrous ( $\text{Fe}^{2+}$ ) states) present moderate kinetics of dissolution.

By decreasing the grain size, the surface exposed to the solution is increased and thus the reaction rate is enhanced. The grain size parameter is particularly important when the mineralized copper is found occluded in a matrix resistant to acid attack.

Given the results shown in Figure 10, a value of  $k_e$  as low as  $8.3 \cdot 10^{-8} [m^3/kg \cdot s]$  that results in the observed low copper recovery could render the leach process to be impractical.

## 5. Conclusions

This paper deals with the transport problem in the copper heap leaching process, where sulfuric acid is the principal leaching agent. The main contribution of this work is the construction of a sensitivity analysis based on changes in relevant physico-chemical parameters and the evaluation of their impact on the performance of the copper extraction process. The main physical parameter considered was the concentration of acid in the irrigation solution applied to the top of the leach heap. The sensitivity analysis shows that this could be an important control factor. Particularly, two chemical parameters were considered one associated with the consumption of sulfuric acid and the other related to the extraction of copper from the solid phase to the liquid phase.

Other parameters of interest to complete the sensitivity analysis are: the height of the heap, porosity and absolute permeability. However, the saturation and velocity of the leach solution were assumed constant throughout the computational experimentation process. Therefore, the next stage of this research is to evaluate the impact of changing these quantities.

## Acknowledgements

E.C. thank to Universidad Católica de Temuco (Convenio de Desempeño DIUCT 2006-2-03).

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